

Index

A

- Abstraction, 94–95
 - mental models, 94–95
 - spatial structuring, 94
- Achievement
 - in geometry, 97–98
- Activities, x
 - All, Some, or None, 211
 - angle, 129–32
 - calculator drawings of prairie plants, 67–76
 - Can You Make It? 113–17, 119–21
 - construction game, 178–80
 - Creating a Mobile, 211, 212–13
 - discovering symmetries, 23
 - folding polygons, 77–87
 - hand drawings of prairie plants, 64–67
 - Hiding Shape Makers, 118–19, 121–23
 - law of sines, 181–83
 - Möbius strips, 34–40
 - Möbius tori, 43–44
 - paper folding to study non-Euclidean geometry, 53–61
 - radians, 183–85
 - spatial development, 142–53
 - Three of These Things Belong Together, 211
 - Triple Play, 211–12, 217
 - You Either Have It or You Don't, 209–10
- Alexandrov gluing, 80
- Alexandrov's theorem, 80–81
- Algebra
 - connections to geometry, 155, 170, 349–65
 - integrated into geometry, 315–32
 - related to geometry formulas, 346
- Algebraic expressions, 297–313
- Algebraic properties, 298, 303–4, 308, 310
- Alignment, *Principles and Standards*
 - principles and textbook, 337, 338, 347
- Angles, ix, 127–38

- activities, 129–32
- alternate interior, 339–40
- classification, 135–36
- concept, 96
- conceptions, 127, 128–29
- estimation, 131
- external on plane curve, 50–52
- external on surface, 53
- inscribed in a circle, 291, 295
- measure, 129–32, 136
- research, 128–29, 133–37
- sum in polygon, 343–46
- sum in quadrilateral, 336, 342–43
- sum in triangle, 336–42
- understanding, 133
- uses, 137
- Apothem, 312
- Archimedean solid, 77
- Area
 - formulas, 297–313; *see also* Rectangle, Triangle, etc.
 - of similar polygons, 263–64
- Assessment, 109, 179–80, 182, 185, 186, 187, 330, 331–32
- Asymmetry, defined, 19
- Attractor, 72–73
- Axiomatizing, 190
- Axioms, ix, xiii

B

- Banach-Tarski paradox, 9
- Biconditional, 193–94
- Bill, Max, 33, 44
- Biogenetic principle, 189
- Bos, Caroline, 45
- Brown, Cameron, 42

C

- Cabri Geometry— *See* Interactive geometry software
- Calatrava, Santiago, 46
- Categorical reasoning, 109–24
- Category membership, 110–12
- Chambord chateau, 45
- Chirality, 43
- Circles, 183–85, 200, 273, 278–79, 280

- excircle, 350, 353–57
 - incircle, 349, 350, 352–57
 - intersections with square, 284–95
 - Classification, 191
 - hierarchical, 94
 - intersections of circle and square, 286–90, 295
 - Coaching, 176–77, 185–86
 - Cognitive conflict, 230
 - Collage theorem, 76
 - Collins, Brent, 44
 - Communication, 213, 215–18, 316; *see also* Vocabulary
 - Concave, 192
 - Concept, 95
 - formal, 95–96
 - natural, 95–96
 - Concept definition, 95
 - Concept image, 199–200
 - Concept learning, 95–96
 - Congruence, 320–23
 - Conjecturing, 116–17, 123, 190, 234, 250–51, 256, 258–59, 273, 283, 284, 287, 323, 324, 335, 338, 347
 - research, 226–31
 - Connected Mathematics Project, 133
 - Connections, 175, 186–87
 - Constructions, 100–101, 178–80, 196–98, 199, 235–49, 256–57, 259–65, 272–73, 274–77
 - angle bisector, 229–30
 - circle and square intersections, 290–95
 - versus drawing, 221–24, 228
 - Continuity, 289
 - Contract
 - for learning and teaching, 329–30
 - Convex, 192
 - Cooperative learning, 177–86, 322–30, 332
 - Counterexamples, 104, 195
 - Curriculum, 142, 144
 - integrated, ix, xii, 315–32
 - teachers' uses, 335–48
 - Curriculum Focal Points for Pre-K–8 Mathematics*
 - recommendations regarding angles, 127
 - Curvature, 49–61
 - Gaussian, 49, 52–53, 55–57
 - negative, 59
 - of curves on Euclidean plane, 50–52, 55
 - of surfaces, 52–55
 - total of a surface, 52, 53, 55, 57, 58
 - Cylinder
 - from Möbius strip, 36–40
- ## D
- Data compression, 76
 - DaVinci, Leonardo, 45
 - Deduction, 123, 235; *see also* Deductive system, Proof
 - among polygon area formulas, 297–313
 - Deductive reasoning
 - research, 226–31
 - Deductive system, 198
 - Defining, 113, 189–201; *see also* Definitions
 - by students, 286–88
 - Defining properties, 94, 96
 - Definitions, ix–x, xii, 92–93, 94, 95, 96, 189–201; *see also* Defining
 - arbitrariness, 190–91, 198, 201
 - benefits, 189, 191
 - convenient, 199
 - correct, 193–94, 200, 201
 - deductive economical, 199
 - economical, 196–99, 200, 201
 - exclusive, 191
 - hierarchical, 191–93, 195, 200
 - inclusive, 191, 195
 - incomplete, 195
 - incorrect, 194, 201
 - inverse-hierarchical-partition, 193
 - not just one, 190–91
 - partitional, 191–93
 - Degree/radian ruler, 183–84
 - Descartes' theorem, 49–55
 - Diagram, 96–97
 - versus figure, 222
 - Discourse, 113, 116–17, 123–24, 175, 184, 284–95
 - Discovery, 76, 176, 177–87, 234, 250–52
 - theorems by students, 269–79
 - Dissemination, 265–66
 - Distance, ix

- concept of, 11–14
 - Euclidean, 11
 - Hamming, 13–14
 - in the plane, formula for, 317–18
 - taxi-cab, 12
- DNA, 45
- Doughnut—See Möbius torus
- Dragging, 100–101, 200, 222, 223, 224
- Drawing
 - versus construction, 221–24, 228
- Drobny, Peter, 44
- E**
- Einstein
 - theory of relativity, 49
- Eisenman, Peter, 45
- English language learners, 169–70
- Environment
 - classroom, 329–30
- Erdős, Paul, 14
- Escher, M. C., 42, 44
- Euclid, ix, 289
 - Elements*, 9, 14, 256–57
 - fifth postulate, 59–61
 - quadrilateral definitions, 192
- Euler phi-function, 43
- Euler's formula, 54
- Exploration, 234, 250–52
- F**
- Fawcett, Harold, xii, 316
- Ferguson, Helaman, 44
- Fermat point, 272–73
 - for quadrilaterals, 240–42
 - for triangles, 236–40
- Fibonacci sequence, 275–76
- Foldability, 79
- Fostering Geometric Thinking in the Middle Grades* professional development program, 158–70
 - attention to teachers' learning, 168–69
 - framework, 161–64
 - materials development, 164–68
- Fractal geometry, 63–76
- Frieze patterns, 22–31
 - crystallographer notation, 30
 - periodic, 27, 29–31

G

- Gauss-Bonnet theorem, 56
- Generalization, 158, 160, 165–67
- Geocadabra, 142–45, 150–52
- Geodesics, 55–58
- Geometers' Sketchpad—See Interactive geometry software
- Geometric Habits of Mind—See also Habits of mind, Mathematical power.
 - balancing exploration and reflection, 161, 163–64
 - criteria, 160
 - defined, 158–60
 - generalizing geometric ideas, 158, 165–68
 - indicators, 161, 162, 163, 164, 165–67
 - investigating invariants, 157–58, 162–63, 165
 - reasoning with relationships, 157, 161–62, 165
- Geometric Supposer, 227–28
- Geometries
 - Euclidean, hyperbolic, elliptic, projective, finite, 11
- Geometry
 - analytic, 67
 - curriculum, 142, 144
 - demonstrative, ix
 - elliptical, spherical, 53, 57, 59, 60, 61
 - etymology, 9
 - Euclidean, 55, 59, 60, 61
 - history, 9–11
 - hyperbolic, 53, 57, 59
 - informal, x
 - integrated course, 315–32
 - non-Euclidean, 49, 52, 53, 59
 - projective, 10
 - standard, 34
 - synthetic, 324–25
 - with coordinates, 315–32
- Geometry's Future conference, xi
- GLaD construction, 274–77
- Golden ratio, 256–58, 260
- Golden rectangle, 260–61
- Graph of a polyhedron, 6
- Graphing calculator, 67–76, 324

for angle activities, 131, 131–32
Group, symmetry, 22

H

Habits of mind, 63, 159–61, 250, 283;
 see also Geometric Habits of Mind
Heron's formula, 351–52
Hilbert, David, 289
Hinged figures, 297–313
Holmes, Sherlock, 28–31
Hyperbolic paper, 58–60

I

Image compression, 63
Inference
 empirical, 93
 logical, 94
Inquiry teaching, 280–81
Instruction, 98, 99, 100, 101, 102, 103,
 109, 123–24, 167–70
 for spatial thinking, 145
Interactive geometry software, 20, 24, 26,
 109–24, 160, 178–80, 195–96,
 199–201, 221–31, 233–52, 253,
 256–65, 267–81
 benefits, 233, 280
 Cabri Geometry, xi, 98, 221, 233,
 267, 279
 Geometer's Sketchpad, xi, 93, 178–
 80, 221, 222, 223, 227, 233–51,
 267, 270, 285, 292–94, 295, 323,
 335, 346, 347
 research, 98–103, 105
 with preservice teachers, 279–80
Invariance, 157–58, 162–63, 165
Isometry, 17, 320
 glide reflection, 18, 24
 reflection, 18, 24
 rotation, 18, 24
 translation, 18, 24
Isosceles trapezoid, 194–95, 199–201,
 255
Iterated function system (IFS), 63, 67,
 72–76

J

Justification, 217, 285, 300, 301–2, 305,
 306, 307, 308, 311; *see also*

Justifying
 research, 100, 101–5
Justifying, 250–51, 316, 324, 340–42;
 see also Justification
 research, 226–31

K

Kite, 100, 195, 196–98, 199, 200
 area formula, 311
 with Shape Makers, 113–17, 118–19
Klee–Chvátal art gallery theorem, 14
Klein bottle, 37
Klein, Felix, 189
Knot theory, 36, 40
Krawczyk, Robert, 46

L

Lakatos, Imre, 288
Language
 challenges, 213–14
 development, 169–70
 for spatial visualization, 145, 146
 teaching strategies, 209–10, 213,
 215–18
Latin cross, 84–86
Law of sines, 181–83
Learning, 91–105
Lesson design, 335–48
Locus, 317–18
Logo, x, 128
 Logo Light, 131–32

M

Mathematical knowledge for teaching,
 168
Mathematical power, 159; *see also* Habits
 of mind
Matrix algebra, 64, 67–69, 74
Max Reinhardt Haus, 45, 46
Metacognition, 164
Minimality
 in definitions, 196, 198
Möbius Band Bridge, 33
Möbius House Het Gooi, 45, 46
Möbius prism, 45, 46
Möbius strip
 applications, 33, 36, 44
 explorations, 34–40

- properties, 34–40
- Möbius torus
 - applications, 44–47
 - explorations, 42–44
 - notation, 42
 - properties, 40–44
- Möbius, August Ferdinand, 36
- Modeling, 233–52, 271

N

- National Science Foundation, xi
- Necessary conditions, 193–95, 196
- Net
 - of rectangular prism, 5–6
- Nets of three-dimensional shapes, 153
- "New math", ix
- Non-Euclidean geometry, 336, 341
- Number theory
 - for Möbius tori, 43

O

- Oblong, 192
- Optimization, 361–63

P

- Paper models, 56–61
- Parallel lines
 - on hyperbolic plane, 59
- Parallelogram, 98–99, 190, 200, 201
 - area formula, 304, 304–6
 - area relation to rectangle, 304
 - centroids of triangles within, 254–56
 - with Shape Makers, 110
- Parent concerns, 331
- Perimeter
 - of similar polygons, 263–64
- Perimeter halving, 82–84
- Periodic
 - defined, 27
- Perrella, Stephen, 45
- Platonic solid, 77, 79
- Pólya, 102, 250, 252, 273
- Polygon
 - convex, 81–84
 - defined, 77
 - interior angle sum, 336, 343–46
 - nonconvex, 84–86

- Polyhedron
 - angle sums, 53–57
 - convex, 77–86
 - defined, 77
 - nonconvex, 86
- Polyomino, 6
- Precalculus, 181
- Preservice teachers
 - secondary school, 335–48
- Principles and Standards for School Mathematics*, 141, 336–37, 338, 342, 348
 - Learning Principle, 176, 336–37, 338, 342
 - recommendations regarding angles, 127
 - Teaching Principle, 175
- Problem posing, 253–266
 - converse, 254–56
 - extension, 254–56
 - generalization, 254–56, 258–59, 266
 - open-ended, 294–95
 - specialization, 254–56
- Problem solving, x, 177–85, 233, 253, 256, 258–59, 266, 316, 331
- Problem-based curriculum
 - implementation challenges, 326–32
 - sample development, 317–26
- Problems
 - area of triangle from midpoints, 226–27
 - as a foundation for later learning, 331
 - construction game, 178–80
 - constructions from sides of a triangle, 259–61
 - cyclic quadrilateral, 278–79
 - equidecomposability, 8–9, 16
 - Fermat's problem for quadrilaterals, 240–42
 - Fermat's problem for triangles, 236–40
 - folding boxes, 5–6, 15
 - folding polygons to polyhedra, 77–87
 - Gamow's treasure, 234–35
 - graphing calculator screen, 4–5
 - graphing quadratic equation's imaginary solutions, 277–78
 - guarding an art museum, 3–4, 15

- Heron's problem, 248–49
 intersections of circles and squares, 284–95
 law of sines, 181–83
 leaf comparison, 12–13
 longest ladder, 357–61
 minimization, 236–48, 272–73
 n -secting sides of triangles, 269–72
 perimeter of triangle with fixed base endpoints, 166–67
 periodic frieze patterns, 30–31
 polygons from centroids of triangles in quadrilaterals, 254–56, 261–63
 polygons inscribed in a circle, 256–58
 quadrilateral with consecutive angle bisectors perpendicular, 101–2
 quadrilateral with two right angles and no parallels, 156, 161, 162, 163, 164
 radii of incircles and excircles, 350–57, 364
 representing polyhedra, 6–7
 robot arm, 7–8, 15
 rugby, 360–63
 spell checker, 13–14
 subdividing a segment, 274–77
 system of roads for a square, 243–46
 two squares form a third, 327–29
 Viviani's problem, 246–48
 Professional development, 330
 Professional development projects
 Fostering Geometric Thinking in the Middle Grades, 155–70
 Project M²: Mentoring Mathematical Minds, 206
 Projective plane, 37
 Proof, x, xii, 19–31, 226–30, 251, 257–58, 260, 263–64, 265, 267–81, 283, 288–95, 298, 316, 322–26, 336, 338–42, 342–343, 343–46; *see also* Proving
 foundation for, 179
 "from the book" (Erdős), 14–15
 issues with diagrams, 97
 midpoint construction, 179–80
 research, 94, 97–98, 100, 103–5
 schemes, 226
 Proportional reasoning, 161
 Prototype effect, 110–12
 Prototypes, 109–24
 Proving 190, 198; *see also* Proof
 Pythagorean theorem, 293, 294, 295, 317–18, 329
- Q**
 Quadratic equation
 imaginary solutions, 277–78
 Quadrilaterals, 190, 191, 194, 196, 201, 205–19, 254–56, 259–64; *see also* Isosceles Trapezoid, Kite, Parallelogram, Rectangle, Rhombus, Square, Trapezoid
 circumcenter, 278–79
 cyclic, 262–63, 278–79, 292
 hierarchy, 255
 interior angle sum, 336, 342–43
 Lambert, 59
 research on students' thinking, 228
 with two right angles, 166
 Questioning, 160, 161–64, 167, 168, 169, 251, 268, 279–80, 285, 291, 299–300, 302, 307–8, 311, 340
- R**
 Radians, 183–85
 Reasoning, 315–32; *see also* Deduction, Justification, Conjecturing
 deductive, 9, 28–30, 93–94
 with relationships, 157, 161–62, 165
 Reconstructive approach, 189–90, 209
 Rectangle, 99, 190, 191, 194, 195, 199, 200, 201
 area formula, 299
 on a hyperbolic plane, 60
 symmetries, 19–22
 Recursion, 346
 Reflection
 as learning process, 316
 Reflections
 in three dimensions, 149–50
 Refutation, 284, 286–90
 Regular polygon
 area formula, 311–13
 area related to parallelogram, 311–12
 area related to rectangle, 312–13
 Representations, 96–97, 294; *see also* Tools
 graphical, 142

symbolic (semiotic), 142
 Representing, 233–52
 Research, xi, 91–105
 interactive geometry software, 221–31
 students' reasoning and proof, 226–31
 students' understanding of figures and properties, 222–25
 Rhomboid, 192
 Rhombus, 190, 192, 193, 195, 198, 199, 200
 with Shape Makers, 119–23
 Robinson, John, 33–34, 44
 Rotation, 128–29, 137

S

Scaffolding, 318, 329
 Segment
 subdividing, 274–77
 Shape Makers—*See* Software
 Similarity, 262–64
 circles, 184
 Smile Math, 131, 132, 136
 Socratic approach, 279–81
 Software—*See also* Interactive geometry software
 Cabri Geometry—*See* Interactive geometry software
 computer algebra system, 350
 constraint-based, 349–50
 construction-based, 349
 Geocadabra, 142–45, 150–52
 Geometer's Sketchpad—*See* Interactive geometry software
 Geometry Expressions, 350
 interactive geometry software, 98, 349–65
 Maple, 350
 Shape Makers, 98–100, 110–24
 symbolic geometry, 349–65
 Soma cubes, 142–50
 Spatial development activities, 142–53
 Spatial operational capacity
 intervention, 144–52
 model, 142–43
 Spatial thinking, 141–53
 uses, 141
 Sphere, 37
 curvature, 56
 topological, 80, 83
 Spiral curriculum, 317–18
 Square, 99, 190, 191, 193, 194, 195, 199, 200, 201, 217
 intersections with circle, 284–94
 with Shape Makers, 110
 Standards, x, 283; *see also* *Principles and Standards for School Mathematics*
 for geometry, 34, 315–16
 Standards-based curriculum, 326
 Steiner, Jacob, 276
 Strategies
 proactive, 224–25
 reactive, 224–25
 Structured exploration process, 158–59
 Students
 anticipating responses of, 291–92
 being challenged, 268
 collaboration by, 123–24
 invention by, xiii, 254–56, 264–66
 Students'—*See also* Understanding
 confidence, 331–32
 engagement, 175–87, 288–94, 316
 engagement in defining, 190
 misconceptions, 19
 ownership of ideas, 284
 role, 329–31
 spatial visualization, 146–50
 theorems proved, 269–79
 thinking, x, xi, xii, 111, 112, 113–23, 156, 169, 284–94, 316
 thinking, interview to probe, 192–93
 thinking, methods to collect, 159
 Sufficient conditions, 193–95, 196
 Symmetry, 17–31, 116–17, 191, 201; *see also* Transformations
 an attribute, 17
 as a reasoning tool, 161–62
 color, 31
 identity, 19, 25–26
 in algebra formula, 352, 360, 364
 of design, 17
 rotational, 320
 type, 21, 27, 30

T

Tangent, 286, 288–89

- Tangram, 8
- Teacher
- role, 123–24, 175–87, 297–99
 - role in curriculum, 336
- Teacher education, x, 155–70, 206, 234–49, 268, 279–81, 335–48; *see also* Professional development regarding angle conceptions, 127–38 secondary school, 253–266
- Teaching
- collaboration, 177, 187
 - establishing norms for proof, 230
 - for conceptual understanding, 175–88, 205–19
 - lesson design, 335–48
- Technology—*See* Interactive geometry software, Graphing calculator, Software
- students decide when to use, 324
- Tetrahedron
- irregular, 81–83
- Textbooks
- by Svenson, 315
 - by Wentworth, 316
 - Discovering Geometry: An Investigative Approach*, 177, 335
 - Phillips Exeter Academy, 316–32
 - teachers' uses, 335–48
- Three-dimensional visualization, 141–53
- Thulaseedas, Jolly, 46
- Thurston, William, 58
- Third International Mathematics and Science Study (TIMSS), 176
- Tools, 195, 290, 295
- Topology, 36
- Torus—*See* Möbius torus
- Transformations, 17–31, 63–76, 162–63, 228, 323
- 180° rotation (half-turn), 25, 26, 30, 298
 - composition, 20
 - glide reflection, 18, 24, 26
 - matrix representation, 68–69
 - of coordinates, 63–64, 67–76
 - reflection, 18, 19, 24, 25, 26, 30, 222, 223
 - rotation, 18, 19, 67, 68–69, 70–72, 225, 297–313
 - scaling, 67–68, 70–72
 - spatial, 142
 - translation, 18, 24, 25, 26, 27, 67, 69, 70–72, 225
- Trapezium, 192
- Trapezoid, 112, 194–95, 199–201, 217; *see also* Isosceles trapezoid
- area formula, 306–10
 - area related to parallelogram, 306–9
 - area related to rectangle, 310
 - definitions of, 206
 - parallelogram as special case of, 101, 255
- Triangle
- ambiguous case, 182–83
 - angle sum, 56–58, 61
 - area, 350–51
 - area formula, 297, 299–304
 - area relation to parallelogram, 304–6
 - area relation to rectangle, 299–304
 - circumcenter, 318
 - congruence, 320–22
 - congruence definition, 198
 - in proof of midpoint construction, 180
 - law of sines, 181–83
 - minimizing distance to vertices, 272–73
 - n -section points of sides, 271
 - on a cube, 56–57
 - on a sphere, 56
 - right triangle formed by sides of regular polygons, 257–58
 - similar, 274–77
 - sum of interior angles, 336–42, 347
 - theorems, 269–72, 272–73
 - trisection points of sides, 269
- Triangulating a polygon, 14–15, 344–45
- Trigonometry, 181–85
- U**
- Understanding, 332
- developing, 321
 - research on, 91–105, 221–31
- Ushio, Keizo, 44
- V**
- van Berkel, Ben, 45
- van Hiele, x, xi
- levels, 91–94, 95, 96, 98, 99–100,

- 101, 105, 133, 134, 135, 136,
137, 206, 207-8, 218, 318-19
- phases, 208, 319-22
- Vatican museum, 45
- Vector, 322, 323
- Vertex, 286, 288-89
- Video, 159
- Visual phenomena, 14
- Visualization—*See* Spatial thinking
- Vocabulary—*See also* Language
 - development, 205-10 213-18, 219
 - spatial, 145, 146

W

- Walter, Marion, 269
- Wilson, Robert, 44
- Word wall, 218
- Writing, 280

Y

Yearbooks

- 1987, *Learning and Teaching
Geometry: K-12*, x
- fifth, *The Teaching of Geometry*, ix
- thirteenth, *The Nature of Proof*, xii
- thirty-sixth, *Geometry in the
Mathematics Curriculum*, ix-x



Contents

Preface	xi
I. Expanding Visions of Geometry.....	1
1. What Is Geometry?	3
Joseph Malkevitch	
<i>City College of New York—York College, Jamaica, New York</i>	
2. Enumerating Symmetry Types of Rectangle and Frieze Patterns: How Sherlock Might Have Done It.....	17
Doris Schattschneider	
<i>Moravian College (Emerita), Bethlehem, Pennsylvania</i>	
3. Möbius Concepts: Strips and Tori.....	33
Yuichi Handa	
<i>California State University—Chico, Chico, California</i>	
David A. James	
<i>University of Michigan—Dearborn, Dearborn, Michigan</i>	
Thomas Mattman	
<i>California State University—Chico, Chico, California</i>	
4. Exploring Curvature with Paper Models	49
Howard T. Iseri	
<i>Mansfield University of Pennsylvania, Mansfield, Pennsylvania</i>	
5. Prairie Plants: Exploring Fractals in Nature	63
Dane Camp	
<i>New Trier Township High School—Winnetka Campus, Winnetka, Illinois</i>	
Erich Hauenstein	
<i>College of DuPage, Glen Ellyn, Illinois</i>	
6. Folding Polygons to Convex Polyhedra	77
Joseph O'Rourke	
<i>Smith College, Northampton, Massachusetts</i>	
II. Learning Geometry	89
7. Highlights of Research on Learning School Geometry	91
Michael T. Battista	
<i>Ohio State University, Columbus, Ohio</i>	

8. Prototypes and Categorical Reasoning: A Perspective to Explain How Children Learn about Interactive Geometry Objects	109
Paul Yu	
<i>Grand Valley State University, Allendale, Michigan</i>	
Jeffrey Barrett	
<i>Illinois State University, Normal, Illinois</i>	
Norma Presmeg	
<i>Illinois State University, Normal, Illinois</i>	
9. Conceptions of Angle: Implications for Middle School Mathematics and Beyond.....	127
Christine Browning	
<i>Western Michigan University, Kalamazoo, Michigan</i>	
Gina Garza-Kling	
<i>Western Michigan University, Kalamazoo, Michigan</i>	
10. Developing the Spatial Operational Capacity of Young Children Using Wooden Cubes and Dynamic Simulation Software	141
Jacqueline Sack	
<i>Rice University, Houston, Texas</i>	
Retha van Niekerk	
<i>University of the Witwatersrand, Johannesburg, Johannesburg, South Africa</i>	
11. Fostering Geometric Thinking in the Middle Grades: Professional Development for Teachers in Grades 5-10	155
Mark Driscoll	
<i>Education Development Center, Inc., Newton, Massachusetts</i>	
Michael Egan	
<i>Augustana College, Rock Island, Illinois</i>	
Rachel Wing DiMatteo	
<i>Education Development Center, Inc., Newton, Massachusetts</i>	
Johannah Nikula	
<i>Education Development Center, Inc., Newton, Massachusetts</i>	

III. Teaching Geometry for Understanding	173
12. Teaching Geometry for Conceptual Understanding: One Teacher's Perspective	175
James Paniati	
<i>Northwest Regional High School, Winsted, Connecticut</i>	
13. Defining in Geometry	189
Michael de Villiers	
<i>University of KwaZulu-Natal, South Africa</i>	
Rajendran Govender	
<i>University of Western Cape, South Africa</i>	
Nikita Patterson	
<i>Kennesaw State University, Kennesaw, Georgia</i>	
14. Advancing Students' Understanding of Quadrilaterals.....	205
Tutita M. Casa	
<i>University of Connecticut, Storrs, Connecticut</i>	
M. Katherine Gavin	
<i>University of Connecticut, Storrs, Connecticut</i>	
15. Using Interactive Geometry Software to Teach Secondary School Geometry: Implications from Research.....	221
Karen F. Hollebrands	
<i>North Carolina State University, Raleigh, North Carolina</i>	
Ryan C. Smith	
<i>North Carolina State University, Raleigh, North Carolina</i>	
16. Representing, Modeling, and Solving Problems in Interactive Geometry Environments	233
José N. Contreras	
<i>University of Southern Mississippi, Hattiesburg, Mississippi</i>	
Armando Martínez-Cruz	
<i>California State University—Fullerton, Fullerton, California</i>	
17. Inventing a Geometry Theorem	253
Armando Martínez-Cruz	
<i>California State University—Fullerton, Fullerton, California</i>	
José N. Contreras	
<i>University of Southern Mississippi, Hattiesburg, Mississippi</i>	

18. Theorems Discovered by Students Inspire Teachers' Development	267
Antonio Quesada	
<i>University of Akron, Akron, Ohio</i>	
19. Using Circle-and-Square Intersections to Engage Students in the Process of Doing Geometry	283
Stephen Blair	
<i>Eastern Michigan University, Ypsilanti, Michigan</i>	
Daniel Canada	
<i>Eastern Washington University, Cheney, Washington</i>	
20. Area Formulas with Hinged Figures	297
Alfinio Flores	
<i>University of Delaware, Newark, Delaware</i>	
21. An Integrated Approach to Teaching and Learning Geometry	315
David Wilson	
<i>State University of New York, Buffalo State College, Buffalo, New York</i>	
22. Redesigning a Traditional Geometry Lesson as an Investigative Activity	335
Jon Davis	
<i>Western Michigan University, Kalamazoo, Michigan</i>	
23. Looking Forward to Interactive Symbolic Geometry	349
Philip H. Todd	
<i>Saltire Software, Tigard, Oregon</i>	

NA PUBLISHING
INC

2009 CONTAINS SOFTWARE (DISC) WHICH
CANNOT BE REPRODUCED. THIS REPRODUCTION
IS MADE FROM THE BEST COPY AVAILABLE.